



Oxford Cambridge and RSA

Tuesday 3 June 2025 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ ms}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- This document has **8** pages.

ADVICE

- Read each question carefully before you start your answer.

1 In this question you must show detailed reasoning.

Vectors **a** and **b** are given by $\mathbf{a} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 3 \\ -2 \end{pmatrix}$.

Determine, in either order

- **a.b**
- **a×b.** [3]

2 You are given that $-7 - 5i$ is one root of the equation $x^3 + 10x^2 + 18x - 296 = 0$.

- (a) Write down another complex root of the equation $x^3 + 10x^2 + 18x - 296 = 0$. [1]
- (b) Using your answer to part (a), express $x^3 + 10x^2 + 18x - 296$ as a product of a real linear factor and a real quadratic factor. [3]

3 Matrices **A and **B** are given by $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ 4 & 8 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -3 & -3 \\ 1 & 2 \end{pmatrix}$.**

- (a) Find the matrix **AB**. [1]
- (b) Verify that $\det(\mathbf{AB}) = \det(\mathbf{A}) \times \det(\mathbf{B})$. [2]
- (c) Use matrices **A** and **B** to demonstrate that matrix multiplication is **not** commutative. [2]

The transformation represented by matrix **A** is denoted by T.

- (d) Show that the point (2, -5) is **not** an invariant point under T. [2]
- (e) Find the matrix which represents the inverse transformation of T. [2]

4 In this question you must show detailed reasoning.

Determine the sum of all cube numbers from 216 to 512 000 inclusive.

[4]

5 In this question you must show detailed reasoning.

(a) Use an algebraic method to determine the two square roots of $-3 + (4\sqrt{7})i$.

Give your answers in the form $a + bi$ where a and b are exact.

[5]

(b) State the relationship between the two arguments of the two square roots found in part (a). [1]

6 One of the regions bounded by two polar curves, C_1 and C_2 , is used to model the face of a flat earring.

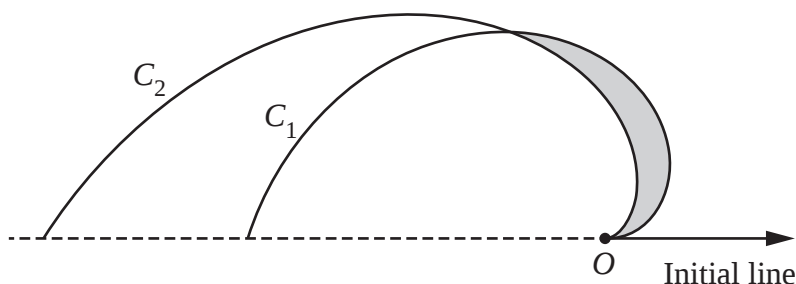
The polar equations of C_1 and C_2 are

$$C_1: r = 2\theta$$

$$C_2: r = \theta^2$$

where $0 \leq \theta \leq \pi$.

The curves C_1 and C_2 are shown in the diagram below with the region used to model the earring shaded.



You are given that C_1 and C_2 intersect at the pole O.

(a) Find the other point of intersection of C_1 and C_2 . Give your answer in polar coordinates. [2]

(b) **In this question you must show detailed reasoning.**

Determine the area of the face of the earring.

[4]

7 In this question you must show detailed reasoning.

(a) Express $\frac{-4x^2 + 5x - 17}{x^3 - x^2 + 3x - 3}$ in partial fractions. **[6]**

(b) Hence determine the exact value of $\int_{\sqrt{3}}^3 \frac{-4x^2 + 5x - 17}{x^3 - x^2 + 3x - 3} dx$. **[4]**

8 A function $f(x)$ is defined by $f(x) = x \sinh 2x$.

(a) Prove by induction that $\frac{d^{2n}f}{dx^{2n}} = 4^n(x \sinh 2x + n \cosh 2x)$ for $n \geq 0$ where $\frac{d^0f}{dx^0}$ is defined as being equal to $f(x)$. **[6]**

(b) Using the formula given in part **(a)**, determine the exact value of the coefficient of x^8 in the Maclaurin series for $x \sinh 2x$. **[3]**

(c) Use the Maclaurin series for e^x to verify your answer to part **(b)**. **[3]**

- 9 When light hits a certain photo-sensitive cell, at time $t = 0$, there is an electrical response in the cell which is denoted by $y(t)$ where both y and t are measured in suitable units. A student wishes to model this response, y .

In an attempt to model y , the student sets up the differential equation

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = 10e^{-3t} \quad (*)$$

which is subject to the following conditions.

- $y = 0$ when $t = 5$
 - $y \geq 0$ for all $t \geq 0$
- (a) By substituting into a suitable differential equation, **verify** that $y = (A + Bt)e^{-3t}$ is a complementary function of the differential equation (*). [2]
- (b) Determine the particular solution for y in terms of t . [8]
- (c) Using your answer to part (b), find the value of y immediately after the light hits the cell. [1]

The cell can be considered to be operating properly if $y < 4$ when $t = 1$.

- (d) Discuss whether the cell can be inferred to be operating properly. [2]

- 10 (a) Find $\frac{1}{2}e^{-u}$ as a percentage of $\sinh u$ for the following values of u .
- $u = 2$
 - $u = 5$
- [1]
- (b) Find, as a percentage of $\sinh u$, the difference between $\sinh u$ and $\cosh u$ for the following values of u .
- $u = 2$
 - $u = 5$
- [1]

A function f is defined for all integers n by $f(n) = \sinh(0.01n) - 5\cosh(0.005n) - 9\tanh n$.

(c) **In this question you must show detailed reasoning.**

With the help of suitable approximations, use an algebraic method to determine the smallest value of n for which $f(n) > 100$. You should **verify** your answer, once found, by direct calculation. You may assume that the required value of n is large.

[6]

END OF QUESTION PAPER

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